Collective excitations and screened interactions in two-dimensional charged Bose systems

B Tanatar† and A K Das‡

† Department of Physics, Bilkent University, Bilkent, 06533 Ankara, Turkey
‡ Department of Physics, Dalhousie University, Halifax, Nova Scotia B3H 3J5, Canada

Abstract. We study the collective excitation spectrum of a two-dimensional charged Bose gas interacting via long- and short-range potentials. The resulting plasmon dispersions depend on the type of interaction. Extending our results to a double-layer system, we calculate the dispersion relations of ensuing optical and acoustic plasmons, and screened interactions. The effective interactions exhibit attractive parts which may have interesting consequences. Comparison of our results with a two-dimensional electron gas is made.

1. Introduction

There has been an intense interest in low-dimensional electron systems in recent years. The discovery of the quantum Hall effect led to a surge of investigations in this area which are still unabated. The discovery of high-temperature superconductivity in layered compounds has further contributed to both theoretical and experimental activities on single-layer and coupled layer electron systems. It may be noted in this connection that a strictly two-layer system has been found to exhibit high-T_c superconductivity. Curiously, the developments in superconductivity research have also revived an interest in a system which received attention prior to the BCS theory of low-T_c superconductivity. This is the charged Bose gas (CBG) which may be regarded as the Bose counterpart of the electron gas (i.e., charged particles obeying Bose statistics). Although studies of the CBG did not lead to the BCS theory, they contributed to our understanding of some important aspects of superconductivity, e.g., the Meissner effect. Early accounts of the CBG in the context of superconductivity may be found in the literature [1]. The CBG was again pursued, long after the BCS theory, as a model many-body system [2]. The ground state energies as a function of the density parameter r_s (see below) and also screening properties of an electron gas and a CBG make an interesting comparison. We remark that although the electron gas has been amply realized in a laboratory, the same cannot be said about the CBG. Injecting deuterium up to a high density into metals like palladium or vanadium may be a promising effort to generate a CBG. In connection with high-T_c superconductivity a two-dimensional Bose gas either with a coulombic interaction [3] or a more complicated gauge field interaction [4] has recently been considered.

Many-body approaches to the various properties of a CBG amount to a wealth of literature [5]. The dielectric properties of a two-dimensional (2D) CBG at finite temperatures were considered by Hines and Frankel [6]. The question of effective screening in a 2D Bose gas with weak interparticle repulsion (dipole interaction) has recently been addressed [7] with application to excitonic systems.

Advances in the growth techniques of semiconducting materials have made the fabrication of double-layer electron systems possible. These novel structures exhibit a number of effects due to interlayer Coulomb interactions [8]. Newly observed fractional quantum Hall states (when a perpendicular magnetic field is applied), interesting transport properties associated with interlayer Coulomb drag, and the possibility of Wigner crystallization are important examples.

In view of the above-mentioned current interests we have been motivated to make a comparative study of two important many-body properties of an electron gas and a CBG in reduced spatial dimensions. The two properties we set out to investigate are (i) collective modes and (ii) screened interactions. In particular, we investigate a single-layer, two-component CBG interacting via long-range Coulomb and short-range contact interaction, and compare our results with those for an electron gas. We then discuss the plasmon dispersions and screened interactions in a double layer of charged bosons.

The plan of the rest of this paper is as follows. In section 2 we discuss the plasmon dispersion relations of various 2D charged Bose systems. Screened interaction in a double-layer CBG and electron gas are compared in section 3. Discussion of our results and a short summary is given in section 4.

2. Collective modes of a 2D charged Bose system

We consider a 2DCBG, and, in analogy with the jellium model of an electron gas, we envisage a uniform neutralizing background. At zero temperature, the system is assumed to be in the condensate phase. The density–density response function for an interacting system of charged bosons within the random phase approximation (RPA) is given by $\chi(q, \omega) = \chi^0(q, \omega)/[1 - v(q)\chi^0(q, \omega)]$, in which the response function for a non-interacting system at $T = 0$ is

$$\chi^0(q, \omega) = \frac{2n\epsilon_q}{(\omega + i\eta)^2 - \epsilon_q^2} \tag{1}$$

with the free-particle energy $\epsilon_q = q^2/2m$, and η a positive infinitesimal quantity. We remark that the above RPA expression [6] is exact at $T = 0$. The bare Coulomb interaction is simply $v(q) = 2\pi e^2/q$ for a 2D system. The plasmon dispersion for a 2DCBG is obtained from the poles of the RPA density–density response function, yielding the Bogoliubov result

$$\omega_{pl}(q) = E_s[x + x^4]^{1/2} \tag{2}$$

with $x = q/q_s$, and $E_s = q_s^2/2m$. Here we have defined the screening wave vector of the Bose condensate $q_s = (8\pi n/a_B)^{1/3}$. Defining a dimensionless density parameter $r_s^2 = 1/(\pi n a_B)$, where a_B is the effective Bohr radius, and n is the 2D density of the boson, we can express the screening wave vector as $q_s a_B = 2/r_s^{2/3}$. Note that the above dispersion law within the RPA behaves like $\omega_{pl} \sim x^{1/2}$ in the long-wavelength limit similar to the 2D electron gas result, and exhibits a free-particle-like behaviour for large wave vectors.

The density–density response function (matrix) for a two-component or two-layer system is given by

$$[\chi(q, \omega)]^{-1} = \begin{pmatrix} [\chi_1^0(q, \omega)]^{-1} - v_{11}(q) & -v_{12}(q) \\ -v_{12}(q) & [\chi_2^0(q, \omega)]^{-1} - v_{22}(q) \end{pmatrix} \tag{3}$$

in the RPA. The RPA takes account of dynamic screening but does not include the corrections due to exchange and correlation effects associated with charge fluctuations in the system. The collective modes of the system are obtained by solving $\det |\chi^{-1}| = 0$.

We first discuss the collective excitations of a single-layer, two-component charged Bose system. In this case, $v_{11} = v_{12} = v_{22} = 2\pi e^2/q$ where we have assumed that particles interact via the long-range Coulomb potential. We also assume equal number density n for both species. The charged bosons only differ in their masses m_1 and m_2, for which we define $\delta = m_1/m_2$. The non-interacting susceptibilities at $T = 0$ are given by $\chi^0_\alpha(q, \omega) = 2n\epsilon_{\alpha q}/[(\omega + i\eta)^2 - \epsilon^2_{\alpha q}]$, where $\epsilon_{\alpha q} = q^2/2m_\alpha$ are the free-particle energies for different species.

The structure of the density–density response function, as given by (3), allows an exact solution for the collective modes. They are obtained from the roots of the quadratic equation for ω^2_{pl}

$$\omega^4_{pl} - \{(1 + \delta^2)x^4 + (1 + \delta)x\}\omega^2_{pl} + [\delta^2 x^8 + \delta(1 + \delta)x^5] = 0. \tag{4}$$

The long-wavelength limit of the collective excitations yields the following plasmon dispersion relations:

$$\begin{aligned}\omega^{(1)}_{pl} &\simeq \sqrt{1 + \delta}\, x^{1/2} \\ \omega^{(2)}_{pl} &\simeq \sqrt{\delta}\, x^2\end{aligned} \tag{5}$$

in which the plasmon energies are in units of $E_s = q^2_s/2m_1$. The screening wave vector for a two-component system is defined as $q^3_s = 8\pi e^2 nm_1$. In figure 1(a) we display the collective modes of a two-component, single-layer charged Bose condensate with mass ratio $\delta = \frac{1}{4}$. The solid lines are the full solutions; upper and lower curves indicate $\omega^{(1)}_{pl}$ and $\omega^{(2)}_{pl}$, respectively. Also shown by dotted lines are the long-wavelength approximations.
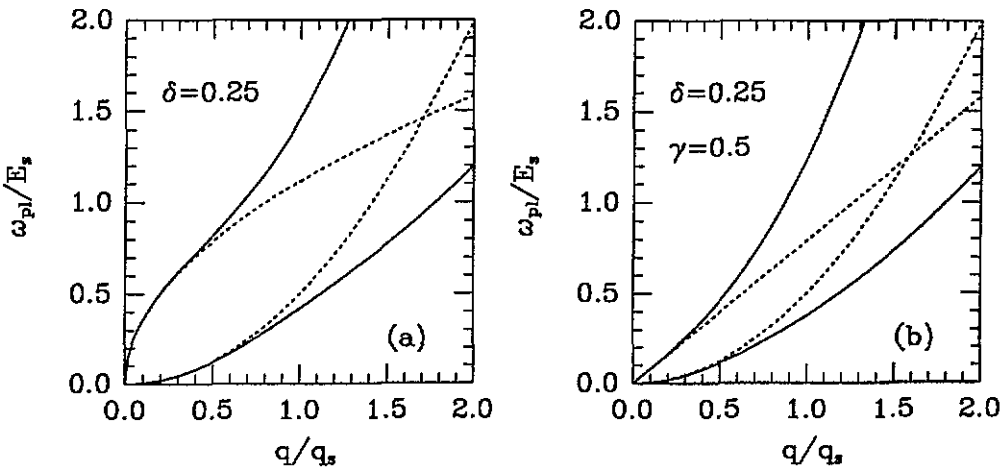
Figure 1. Collective modes of a single-layer, two-component Bose system with mass ratio $\delta = \frac{1}{4}$. (a) Collective modes (solid lines) in a charged Bose gas interacting via Coulomb potential. (b) Collective modes (solid lines) of a Bose system interacting with a short-range potential of strength $\gamma = \frac{1}{2}$. In both cases, dotted lines indicate the long-wavelength approximation to the exact collective excitations.

It is of interest to see how the interparticle interaction affects the collective modes. We have already considered a long-range (Coulomb) interaction. We now consider a two-component, single-layer Bose system where the particles interact via a short-range (in fact, a zero-range), contact potential. This means that $v_{11} = v_{12} = v_{22} = v_0$, where v_0 is constant.

Defining a dimensionless quantity $\gamma = 2nv_0/E_s$, collective excitations are obtained from the solution of

$$\omega_{pl}^4 - [(1 + \delta^2)x^4 + \gamma(1 + \delta)x]\,\omega_{pl}^2 + [\delta^2 x^8 + \delta\gamma(1 + \delta)x^6] = 0 \qquad (6)$$

from which we obtain in the long-wavelength limit

$$\begin{aligned}
\omega_{pl}^{(1)} &\simeq \sqrt{\gamma(1 + \delta)}\, x \\
\omega_{pl}^{(2)} &\simeq \sqrt{\delta}\, x^2.
\end{aligned} \qquad (7)$$

Figure 1(b) shows the collective modes of a two-component, single Bose system interacting via a short-range potential. The solid lines represent the full solutions; the upper and lower curves indicate $\omega_{pl}^{(1)}$ and $\omega_{pl}^{(2)}$, respectively. The dotted lines represent the long-wavelength approximations. It is interesting to note that the second mode $\omega_{pl}^{(2)}$ in both cases (long range and short range) has the same dispersion, which is free-particle-like, $\omega_{pl} \sim x^2$, whereas the first collective mode $\omega_{pl}^{(1)}$ depends on the interaction type. We point out that in the corresponding system of a two-component electron gas layer interacting via Coulomb potential, the second plasmon mode displays [9] an acoustic dispersion, i.e., $\omega_{pl} \sim x$.

We believe that the difference between the dispersion characteristics of the collective modes for fermion and boson systems arises due to statistics (for the same interparticle interaction) and multicomponent nature. For a single-particle system one does not find this difference. It may be recalled that the dispersion characteristics are a consequence of the phase space restriction. Statistics partly contributes to this restriction. Now, for multicomponent systems the collective mode dynamics is more involved and the effect of statistics is also present. Therefore we find the difference for multicomponent systems. It should be noted that for $m_1 = m_2$ the second solution in equation (5) should be discarded because we than have an effectively one-component system.

In order to explore further the argument that the difference in dispersion characteristics is due to a combination of statistics and multicomponent nature, we briefly discuss our results in connection with the case of a classical two-component plasma. Vignale [9] has shown that a two-component electron liquid (in which there are light and heavy electrons interacting via Coulomb potential) has collective modes $\omega_{pl}^{(1)} \sim q^{1/2}$, and $\omega_{pl}^{(2)} \sim q$ in the long-wavelength limit. We observe that the influence of statistics is reflected in the second mode. In the Fermi statistics (electron gas), the second plasmon mode is acoustic-like, whereas in the Bose statistics it is free-particle-like. In a classical two-component plasma (at $T = 0$) there is only one collective mode which is given by $\omega_{pl}^2 = \omega_1^2 + \omega_2^2$, where ω_1 and ω_2 are individual plasma frequencies. Thus in the appropriate limit, the first collective modes of two-component Fermi and Bose systems coincide with the classical result.

Having considered a single-layer system, we now move on to a double-layer charged Bose gas system. A double layer is a charge-separated system and may also not viewed as a strictly 2D system. We first look at two identical layers with the same number density n. The intralayer Coulomb interaction is $v_{11} = v_{22} = 2\pi e^2/q$, whereas the interlayer interaction is $v_{12} = v_{21} = v_{11}\,e^{-qd}$, in which d is the interlayer separation. The collective modes of a double-layer system are given by [10]

$$\omega^{(1,2)} = [x(1 \pm e^{-x\tilde{d}}) + x^4]^{1/2} \qquad (8)$$

where $\tilde{d} = dq_s$. In the long-wavelength limit the plasmon dispersions are

$$\begin{aligned}
\omega_{pl}^{(1)} &\simeq \sqrt{2}\, x^{1/2} \\
\omega_{pl}^{(2)} &\simeq \sqrt{\tilde{d}}\, x.
\end{aligned} \qquad (9)$$

The first of these modes is labelled the optical plasmon, and the second one the acoustic plasmon. If we now consider the more general case of two non-identical layers with different densities and particle masses, the resulting equation that yields the collective modes is

$$\omega_{pl}^4 - [(1 + \delta^2)x^4 + (1 + \delta/\gamma)x]\,\omega_{pl}^2 + [\delta^2 x^8 + \delta(1/\gamma + \delta)x^5 + (1 - e^{-2x\bar{d}})\delta x^2/\gamma] = 0.$$

$$(10)$$

The long-wavelength plasmon excitations have the form

$$\omega_{pl}^{(1)} \simeq (1 + \delta/\gamma)^{1/2}\,x^{1/2}$$

$$\omega_{pl}^{(2)} \simeq [2\delta\bar{d}/(\gamma + \delta)]^{1/2}\,x \tag{11}$$

where we have used $\delta = m_1/m_2$ and $\gamma = n_1/n_2$, and also assumed that $x\bar{d} \ll 1$. We note that optical–acoustic plasmon identification of the modes persist, similar to the analogous case in double-layer electron systems [11]. We show in figure 2 the full (solid lines) and long-wavelength forms (dotted lines) of collective excitations of a two non-identical layers of a Bose condensed system with mass ratio $\delta = \frac{1}{4}$. The upper and lower curves indicate $\omega_{pl}^{(1)}$ and $\omega_{pl}^{(2)}$, respectively. We took for illustration purposes $\gamma = 1$ and $\bar{d} = 1$.
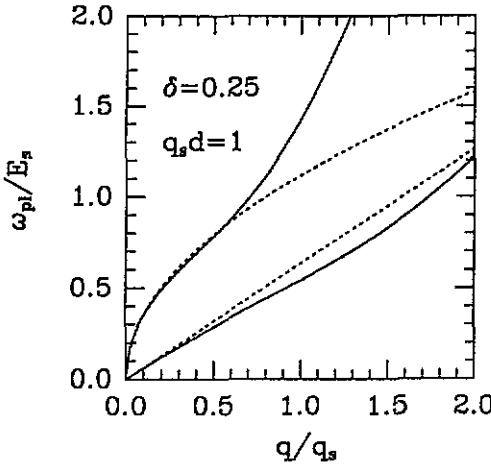

Figure 2. Collective modes of a double-layer, two-component charged Bose gas, for the mass ratio $\delta = \frac{1}{4}$, and layer separation $q_s d = 1$. The upper and lower curves (solid lines) indicate the optical and acoustic plasmons, respectively. The dotted lines are the long-wavelength approximation.

3. Screened interactions

We now turn our attention to the screened interactions in double-layer systems. We shall be concerned only with static interactions. For a two-layer system (Bose or Fermi) the screened interactions may be written in matrix notation

$$v_{ij}^{sc}(q) = \sum_k v_{ik}(q) \left[\frac{1}{\varepsilon(q)}\right]_{kj} \tag{12}$$

where the elements of the static dielectric function are defined as $\varepsilon_{ij}(q) = \delta_{ij} - v_{ij}(q)\,\chi_i^0(q)$. Here the static susceptibility is related to the zero-frequency limit of the dynamic

susceptibility, i.e., $\chi^0(q) = \chi^0(q, \omega = 0)$. For non-identical layers there are four distinct screened interactions (two interlayer and two intralayer). For two identical charged Bose layers, we have the screened interlayer interaction given as

$$v_{12}^{sc}(q) = \frac{v_{12}(q)}{[1 - v_{11}(q)\chi_1^0(q)]^2 - [v_{12}(q)\chi_1^0(q)]^2} \tag{13}$$

and the intralayer screened interaction given as

$$v_{11}^{sc}(q) = \frac{v_{11}(q) - [v_{11}^2(q) + v_{12}^2(q)]\chi_1^0(q)}{[1 - v_{11}(q)\chi_1^0(q)]^2 - [v_{12}(q)\chi_1^0(q)]^2}. \tag{14}$$

Scaling all length parameters with the screening wave vector q_s, we obtain the following expressions for the static response functions in a two-layer charged Bose system within the RPA:

$$\varepsilon_{11}(q) = 1 + 1/x^3$$
$$\varepsilon_{12}(q) = e^{-x\tilde{d}}/x^3. \tag{15}$$

The real space expressions for the screened interactions are obtained by Fourier transformation,

$$V_{12}^{sc}(r) = (e^2 q_s) \int_0^\infty dx \, \frac{x^6 e^{-x\tilde{d}} J_0(x\tilde{r})}{(1 + x^3)^2 - e^{-2x\tilde{d}}} \tag{16}$$

$$V_{11}^{sc}(r) = (e^2 q_s) \left\{ \frac{1}{\tilde{r}} - \int_0^\infty dx \, \frac{(1 + e^{-x\tilde{d}}) J_0(x\tilde{r})}{1 + x^3 + e^{-x\tilde{d}}} \right\} \tag{17}$$

where $J_0(x)$ is the zeroth-order Bessel function of the first kind. Note that in the above expressions as $\tilde{d} \gg 1$, V_{11}^{sc} approaches the single-layer result of Hines and Frankel [6]. We show in figure 3 the screened interactions for a two-layer Bose condensate. The intra- and interlayer interactions are depicted by solid and dashed lines, respectively. Also shown for comparison (dotted line) is the bare Coulomb potential. We observe that unlike the Coulomb $\sim 1/r$ potential, the screened interactions exhibit a short-range attractive part. The attractive potential of the intralayer interaction is largely independent of the layer separation, whereas the interlayer interaction decreases in magnitude as $q_s d \gg 1$. We also note that the interlayer screened interaction remains finite at $r = 0$, in contrast to the $1/r$ singularity in the intralayer interaction. Hines and Frankel [6] have shown that the statically screened interaction in a single-layer system behaves as $\sim 1/r^7$ for large distances. We surmise that our intralayer interaction $V_{11}^{sc}(r)$ should exhibit a similar behaviour.

In the case of a two-layer electron system the static dielectric functions are given by

$$\varepsilon_{11}(q) = 1 + (r_s/\sqrt{2}) f(x)$$
$$\varepsilon_{12}(q) = e^{-x\tilde{d}} (r_s/\sqrt{2}) f(x) \tag{18}$$

where $f(x) = 1/x - \theta(x - 1)\sqrt{x^2 - 1}/x^2$, in which $x = q/2k_F$ and $\theta(x)$ is the unit step function. The screened interactions for a double-layer electron gas has been considered by Zheng and MacDonald [12], Szymański *et al* [13], and Cordes and Das [14]. The screened interactions have the same formal expressions as those for charged bosons, and their Fourier transform gives

$$V_{12}^{sc}(r) = (2k_F e^2) \int_0^\infty dx \, \frac{e^{-x\tilde{d}} J_0(x\tilde{r})}{[1 + r_s f(x)/\sqrt{2}]^2 - [e^{-x\tilde{d}} r_s f(x)/\sqrt{2}]^2} \tag{19}$$
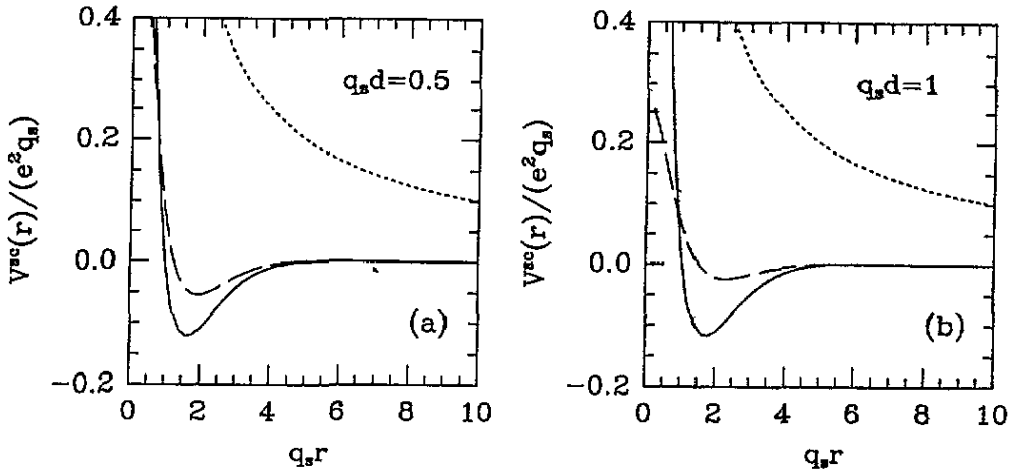
Figure 3. Statically screened intralayer (solid lines) and interlayer (dashed lines) Coulomb interactions in a double-layer charged Bose gas, for layer separations (a) $q_s d = 0.5$, and (b) $q_s d = 1$. Dotted lines indicate the bare Coulomb potential.
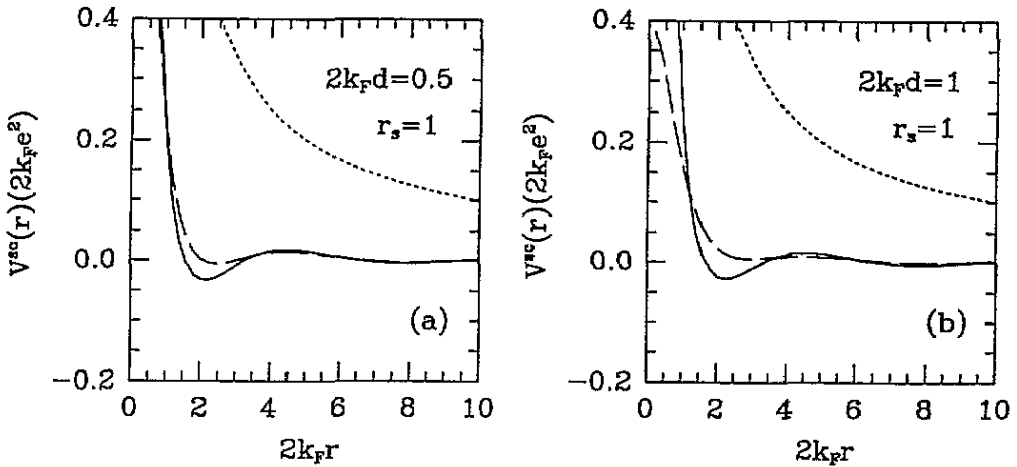

Figure 4. Statically screened intralayer (solid lines) and interlayer (dashed lines) Coulomb interactions in a double-layer electron gas, at $r_s = 1$, and layer separations (a) $2k_F d = 0.5$, and (b) $2k_F d = 1$. Dotted lines indicate the bare Coulomb potential.

$$V_{11}^{sc}(r) = (2k_F e^2) \left\{ \frac{1}{\tilde{r}} - \int_0^\infty dx \, \frac{(1 + e^{-x\tilde{d}}) J_0(x\tilde{r})}{1 + [\sqrt{2}/r_s f(x)] + e^{-x\tilde{d}}} \right\}. \tag{20}$$

In the above expressions $\tilde{r}$ and $\tilde{d}$ are quantities in units of $2k_F$. We show for comparison in figure 4 the screened inter- and intralayer interactions for a double-layer electron gas by dashed and solid lines, respectively. The dotted curve indicates the bare Coulomb interaction. The screened interactions have certain noteworthy features, and the Bose and Fermi cases (figures 3 and 4, respectively) make an interesting comparison. Each of the screened potentials develops an attractive well which is deeper for the intralayer interactions. Note that the attractive wells (for either intralayer and interlayer) are stronger in the Bose gas than in the Fermi case. However in the Fermi (i.e., electron gas) case the $V^{sc}(r)$ develop

weakly oscillatory tails. We believe these are Friedel-type oscillations (related to the Fermi surface). The oscillatory tails may be weaker than in the pure 2D case. It is also to be noted that for a weaker interlayer coupling ($q_s d = 1$ and $2k_F d = 1$) the $V^{sc}(r)$ seem to lose their hard-core character. The above features of the screened interactions may have relevance to superconductivity in layered compounds.

4. Discussion

In contrast to the corresponding case of 2D electron gas, there is no Landau damping within the RPA for the charged Bose gas as may easily be seen from (1). Damping may be induced by disorder or thermal scattering. The finite-temperature dielectric function of a 2D charged Bose gas has been considered by Hines and Frankel [6] and recently by Kachintsev and Ulloa [7]. At $T = 0$, damping will arise from higher-order diagrams [5] (beyond RPA) for the polarizability $\chi(q, \omega)$.

   We have based our treatment of the collective mode spectrum of a 2D charged Bose gas on the RPA. This approach when applied to the high-density electron gas has been found to describe the dielectric properties quite well. As the density is lowered in the charged particle systems, corrections to the RPA become important. A convenient way of accounting for such corrections is through the local field factor $G(q)$ in the static approximation. It takes the exchange and correlation effects into account, and may be incorporated within our theoretical scheme by making the replacement $v_{ij}(q) \rightarrow v_{ij}(q)[1 - G_{ij}(q)]$. The effects of local field corrections on the plasmons in a charged Bose gas have been studied by Gold [10]. It was pointed out by Gold [10] that a roton-like structure appears in the plasmon dispersion with the inclusion of local field factor $G(q)$. The role of static local field corrections in a 3DCBG, comparing various approximation schemes, has recently been investigated by Conti *et al* [15].

   We have considered a 2D charged Bose gas at $T = 0$, where the system is in the condensate phase. The depletion of the condensate at low temperature due to interactions was calculated using renormalization group techniques [16]. From a different point of view, we might take the distribution function of the charged Bose gas to be [17]

$$f(k) = n_0 \, \delta(k) + N(\epsilon_k, \mu)$$

where the first term describes the particles in the condensate (temperature independent), and the second term describes the non-condensed particles. $N(\epsilon_k, \mu) = [e^{\beta(\epsilon_k - \mu)} - 1]^{-1}$ is the momentum distribution of the non-condensed particles at a given temperature T, and chemical potential μ. Hines and Frankel [6] and Kachintsev and Ulloa [7] use this second part of the momentum distribution to investigate certain properties of the dielectric function of 2D charged bosons. Note that the imaginary part of the collective mode that Hines and Frankel [6] discuss is due to thermal effects only.

   In summary we have studied the collective mode dispersions of a 2D, single- and double-layer charged Bose gas. In the case of a two-component single-layer system, we have found that the collective excitations display behaviour different from the corresponding electron system. A double-layer CBG, on the other hand, exhibits plasmon dispersions similar to those of an electron gas. We have considered the screened interactions within the RPA using the static dielectric function. Statically screened interactions of a double-layer CBG show a marked difference from the bare Coulomb interaction. Our results could be helpful in distinguishing the Fermi liquid and Bose liquid models of high-T_c superconductivity.

Acknowledgments

References

[1] Schafroth M R 1960 *Solid State Physics* vol 10 (New York: Academic)
Blatt J M 1964 *Theory of Superconductivity* (New York: Academic)
[2] Fetter A L 1971 *Ann. Phys.* **64** 1
[3] Alexandrov A S and Mott N F 1993 *Phys. Rev. Lett.* **71** 1075; 1994 *Rep. Prog. Phys.* **57** 1197
[4] Feigelman M V, Geshkenbein V B, Ioffe L B, and Larkin A I 1993 *Phys. Rev.* B **48** 16641
[5] We cite a few examples:
Lee J C 1975 *Phys. Rev.* B **12** 2644
Caparica Á A and Hipólito·O 1982 *Phys. Rev.* B **26** 2832
[6] Hines D F and Frankel N E 1979 *Phys. Rev.* B **20** 972
[7] Kachintsev D M and Ulloa S E 1994 *Phys. Rev.* B **50** 8715
[8] Suen Y W, Engel L W, Santos M B, Shayegan M and Tsui D C 1992 *Phys. Rev. Lett.* **68** 1379
Eisenstein J P, Boebinger G S W, Pfeiffer L N, West K W and He S 1992 *Phys. Rev. Lett.* **68** 1383
Gramila T J, Eisenstein J P, MacDonald A H, Pfeiffer L N and West K W 1991 *Phys. Rev. Lett.* **66** 1216
Swierkowski L and Neilson D 1991 *Phys. Rev. Lett.* **67** 240
[9] Vignale G 1988 *Phys. Rev.* B **38** 811
[10] Gold A 1992 *Z. Phys.* B **89** 1 and references therein
[11] Das Sarma S and Madhukar A 1981 *Phys. Rev.* B **23** 805
Santoro G E and Giuliani G F 1988 *Phys. Rev.* B **37** 937
[12] Zheng L and MacDonald A H 1994 *Phys. Rev.* B **49** 5522
[13] Szymański J, Świerkowski L and Neilson D 1994 *Phys. Rev.* B **50** 11002
[14] Cordes J G and Das A K 1991 *Solid State Commun.* **80** 145; unpublished
[15] Conti S, Chiofalo M L and Tosi M P 1994 *J. Phys.: Condens. Matter* **6** 8795
[16] Fisher D S and Hohenberg P C 1988 *Phys. Rev.* B **37** 4936
Chang C and Friedberg R 1995 *Phys. Rev.* B **51** 1117
[17] Szépfalusy P and Kondor I 1974 *Ann. Phys.* **82** 1
Griffin A 1993 *Excitations in Bose-Condensed Liquids* (Cambridge: Cambridge University Press)